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**Reduced-Order Modeling Method for Fatigue Life Predictions of
Hybrid Electric Vehicle Batteries**

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ABSTRACT

The goal of this work is to develop an efficient numerical modeling method for the structural dynamic response of hybrid electric vehicle (HEV) batteries in order to support fatigue life predictions. The dynamics of HEV battery packs are known to feature very high modal density in many frequency bands. The high modal density combined with small, random structural variations among the cells (which are unavoidable in practice) can lead to drastic changes in the structural dynamics. Therefore, it may be important to perform probabilistic simulations of the structural dynamic response with cell-to-cell parameter variations in order to accurately predict the fatigue life of a battery pack. However, the computational time for obtaining forced response results for just a single sample of parameter variations with a finite element model can be on the order of a day. One approach to overcome this challenge is to generate parametric reduced-order models (PROMs). The novel approach is based on two key assumptions. First, it is assumed that the mode shapes of a battery pack (with parametric variations in the cells) can be represented by a linear combination of the mode shapes of the nominal system (with identical cells). Second, it is assumed that the frame holding each cell has vibratory motion. PROMs are validated numerically with full-order finite element models by comparing forced response predictions. The new PROMs are able to predict the dynamics of battery packs 1,000 to 10,000 times faster than full-order finite element models while maintaining accuracy. For the few initial cases considered, small cell-to-cell parameter variations are found to lead to an increase of up to 60% in the vibration amplitude of a battery cell, which could have a significant impact on fatigue life.

INTRODUCTION

A typical hybrid electric vehicle (HEV) battery has 100–300 cells, which are stacked into several packs. Because these cells are nominally identical, battery packs fall under the class of structures known as periodic structures. The dynamics of periodic structures are known to feature very high modal density in many frequency bands. The high modal density combined with small, random structural variations among the cells (which are unavoidable in practice) can lead to drastic consequences on the structural dynamics. Therefore, it may be important to use statistical dynamic response calculations for predicting the fatigue life of a pack. Such statistical calculations are hard to perform using linear methods, because the mode shapes of a pack depend in a nonlinear fashion on the parameters of each cell. The alternative is to use sample-based statistical analyses.

However, typical finite element models (FEMs) of battery packs have several million degrees of freedom (DOF). Thus, the computational time for obtaining just a single sample can be on the order of a day.

To overcome this issue, in the field of structural dynamic analysis, component mode synthesis (CMS) [1-7] is well established as an alternative to conventional FEMs with large numbers of DOF. CMS belongs to a wide class of domain decomposition techniques. CMS divides the global structure into several substructures, and the DOF of each individual substructure are reduced significantly. Then, the substructures are reconnected, and the dynamic response of the system is predicted very efficiently and accurately. However, classical CMS must be modified in order to account for parametric variability in the structure. Thus, alternate, design-oriented techniques have been developed.

One such approach is to generate what is referred to as parametric reduced-order model (PROM). PROMs were introduced initially by Balmès [8-9] to avoid the expensive process of reanalysis of complex structures. In addition, several other PROM methods have been developed [10-13]. In particular, the multi-component PROM (MC-PROM) technique has been developed recently by Hong et al. [13]

These PROM techniques are highly efficient methods for estimating the statistics of the structural dynamic response. However, for a structure with very high modal density, previously developed PROMs have to be modified to efficiently capture the dynamic response. In particular, the component mode mistuning (CMM) [14] method was developed for predicting the dynamic response of bladed disks found in turbomachinery rotors. Typically these rotors suffer from high modal density. Thus, small structural variations in the blades affect significantly the system-level dynamic response. Nonetheless, it has been shown that the mode shapes of a mistuned bladed disk can be represented as a linear combination of the mode shapes of the tuned bladed disk. [15] This allows CMM to capture the dynamic response effectively with a small number of DOF, and that is the inspiration for the new PROMs, as discussed next.

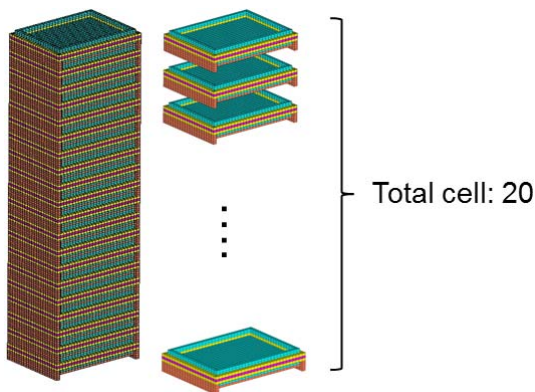


Figure 1: The geometry of a battery pack with 20 cells

STRUCTURAL PROPERTIES OF BATTERY PACKS

HEV battery packs typically have 100-300 individual cells that are nominally identical. To demonstrate the structural characteristics of battery packs, an academic battery model was developed using finite elements as shown in Figure 1. A total of 20 nominally identical cells are stacked.

Figure 2 shows a single cell and the frames that join it to the adjacent cells. The single cell is a plate-like structure which can have structural variations in its density (ρ) or elastic modulus (E). The repetitive cells are mechanically coupled through the frames and that induces a high modal density to the entire battery pack structure, as shown in Figure 3.

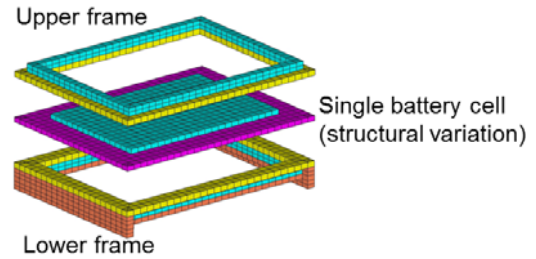


Figure 2: The geometry of a single battery cell

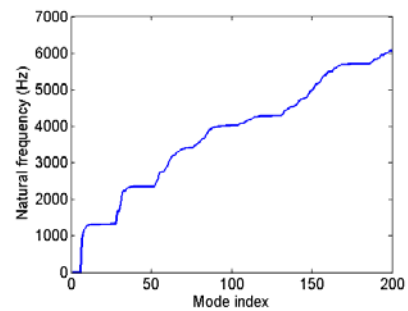


Figure 3: Natural frequencies of the academic battery pack

Table 1: Two cases of Young's modulus variations

Case 1		Case 2	
Cell	Variation	Cell	Variation
1	+5%	3	+3%
5	-7%	9	-5%
12	+1%	13	+2%
16	+3%	20	-5%

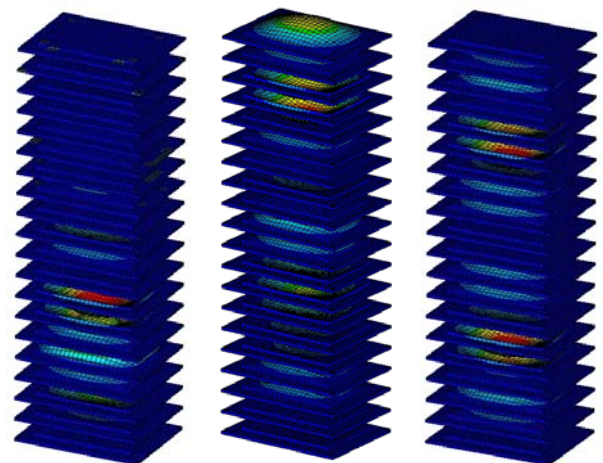


Figure 4: Mode shapes of the nominal structure with identical cells (left) and the structures with Case 1 (center) and Case 2 (right) of cell parameter variations

The flat regions in Figure 3 indicate frequency ranges of high modal density. For example, there are over 10 modes in the range 1,310–1,320 Hz. If the battery pack had more cells, the modal density would be even higher.

To examine how structural variations in the cells affect the structural response, we applied the elastic modulus variations described in Table 1, and compared the mode shapes of the structure with nominal parameters (no variation) and the mode shapes of the structure with Cases 1 and 2 of variations as shown in Figure 4. In general, all cells have some variability. To observe the consequences of small parameter variations, we first applied variations to only 4 cells at a time.

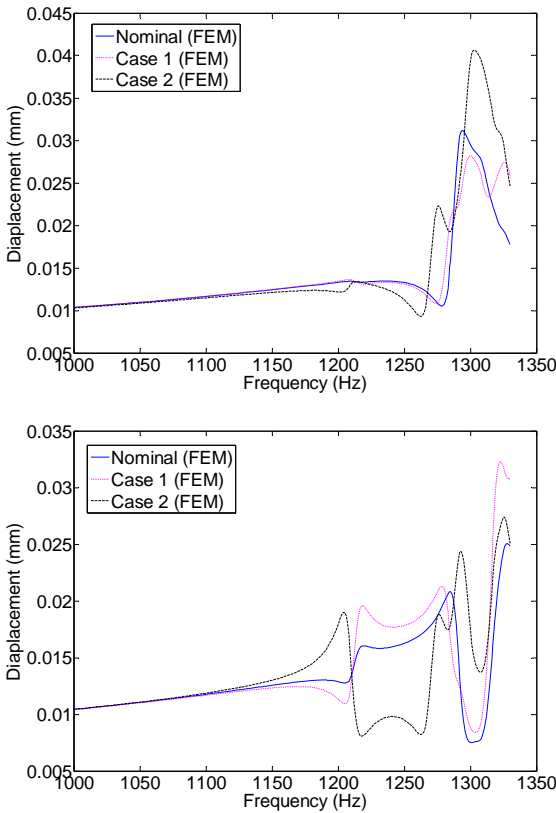


Figure 5: Forced response for the 10th cell (top) and 18th cell (bottom) of for the nominal case of no parameter variations versus two cases with parameter variations

Although the structural variations are small, the displacements are affected significantly. In particular, note that some displacements are localized at a few cells for the case in which there are no parameter variations. In addition, the forced responses were compared for each case of variation as shown in Figure 5. These results also show that small variations in local parameters can induce large changes in the global response.

Figure 5 shows the response of the 10th cell (top plot) and the 18th cell (bottom plot). As shown in Table 1, there are no variations in the parameters of cells 10 and 18. However, there are significant changes in the dynamic response of these cells. For example, the maximum response of the 10th cell of the battery with nominal parameters is 0.03 mm, whereas the maximum response of the same cell in Case 2 of parameter variations is 0.04 mm. The maximum parameter variation between the nominal battery and that of Case 2 is only 5%. Nonetheless, the variation in the maximum response is almost 35%. This demonstrates that small local structural variations can have large global consequences. To capture the dynamic characteristics, we developed the new approach described in the next section.

METHODOLOGY

The equations of motion (EOMs) for the structure with no variation and with variation can be expressed as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}, \quad (1)$$

$$(\mathbf{M} + \mathbf{M}^\delta)\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + (\mathbf{K} + \mathbf{K}^\delta)\mathbf{x} = \mathbf{F}, \quad (2)$$

where \mathbf{M}^δ and \mathbf{K}^δ are the mass and stiffness variations due to the structural variations. Based on Equations (1) and (2), the mode shapes are defined by the following eigenvalue problems

$$\mathbf{K}\Phi^t - \mathbf{M}\Phi^t\Lambda^t = \mathbf{0}, \quad (3)$$

$$(\mathbf{K} + \mathbf{K}^\delta)\Phi^m - (\mathbf{M} + \mathbf{M}^\delta)\Phi^m\Lambda^m = \mathbf{0}, \quad (4)$$

where superscript t and m indicate the tuned (nominal) and mistuned (structural with variation) quantities.

The novel approach is based on two key assumptions. The first assumption is that the mode shapes Φ^m of a pack with parametric variations can be approximated as a linear combination of the mode shapes Φ^t of nominal pack with no parametric variations. This first assumption is ensured by the high modal density.

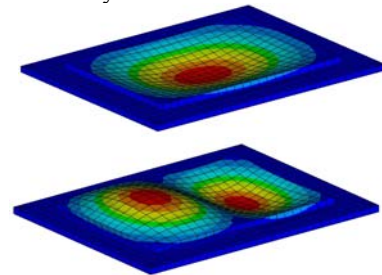


Figure 6: 1st and 2nd modes of a fixed-boundary cell

The second assumption in the PROMs is that the variations in mass and stiffness of a cell can be projected onto a small set of modes of the nominal cell with a fixed boundary, as shown in Figure 6. This second assumption usually relies on

the fact that the boundary motions can be ignored because the boundary of the plate is not moving much.

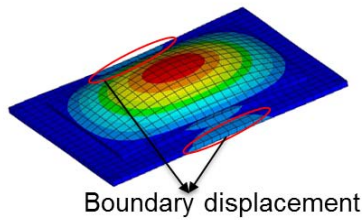


Figure 7: Boundary-displaced motion of a plate-like cell

However, it turns out that the boundary motion has to be considered because it is not small, as shown in Figure 7. Thus, the plate-like modes of a nominal cell—with its boundary displaced the same amount as the frame—are used in the proposed PROMs. This is a key step for ensuring accuracy. This approach is distinct from the usual CMM method. For example, the CMM method does not account for the boundary motion.

By combining these two key assumptions, PROMs are developed and their effectiveness is demonstrated for the academic battery pack structure.

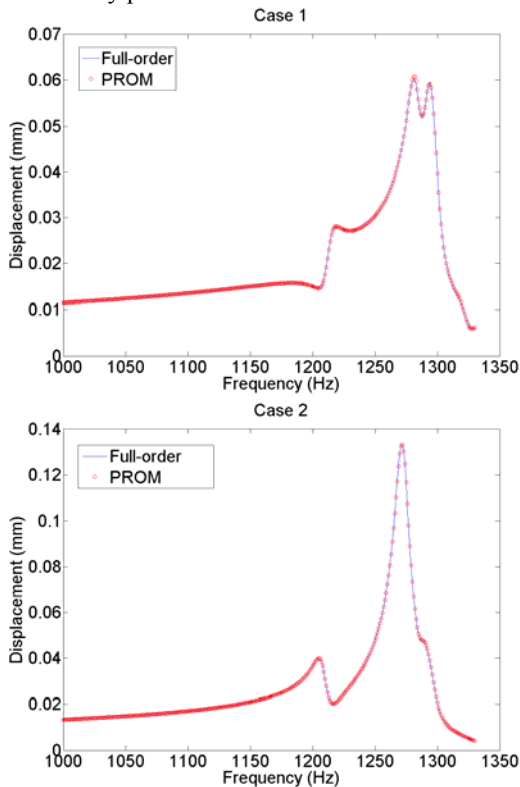


Figure 8: Forced response predictions for the 20th cell predicted by a full-order finite element model and a PROM for parameter variation Case 1 (top) and Case 2 (bottom)

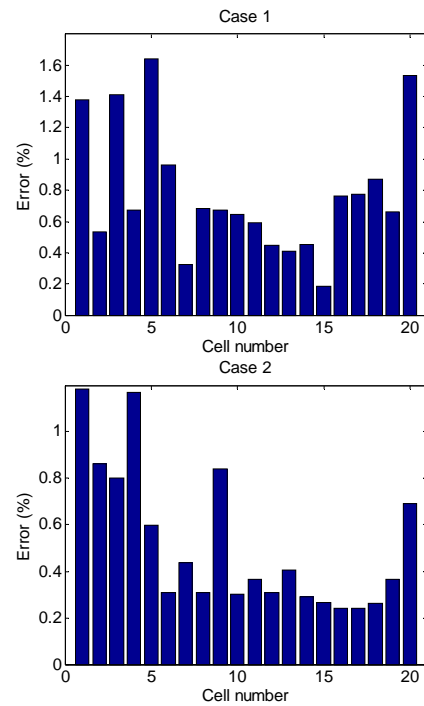


Figure 9: Maximum errors for each cell in the frequency range of interest for cases 1 (top) and 2 (bottom)

NUMERICAL RESULTS

Numerical results to demonstrate the performance of new method have been obtained using an academic battery model shown in Figure 1. This academic model has 208,753 DOF and 20 nominally identical cells. The frequency range of interest is 900-1,950 Hz (the first flat region in Figure 3). Two cases of Young's modulus variations were applied to 4 cells in the pack, as shown in Table 1. Forcing was applied at the center points of each cell. A PROM was constructed and used to compute the response at those locations.

The PROM predictions agree very well with predictions of the FEM. For example, Figure 8 shows the response of the 20th cell for two cases of variations. Figure 9 shows the maximum error between the PROM and the FEM predictions for all cells over the entire frequency range of interest. The maximum errors are between 1.18% and 1.64%. The analysis time required by the PROM for each variation is about 9,000 times shorter than that of the FEM.

This computational gain is expected to be even larger for more refined models. That is because the PROM captures the low-dimensional physics of the problem. This low dimensionality means that only a few coordinates are necessary to describe the dynamics of the actual physical system. This number of coordinates is a feature of the physics, not of the model used to discretize the physics. The model can increase in size by mesh refinement. However,

the physics remains the same and require only a few coordinates. The key is to find these coordinates, and PROM techniques are intended to do just that. Thus, the size of the PROM is not expected to increase when the size of the full order model increases (e.g., by mesh refinement).

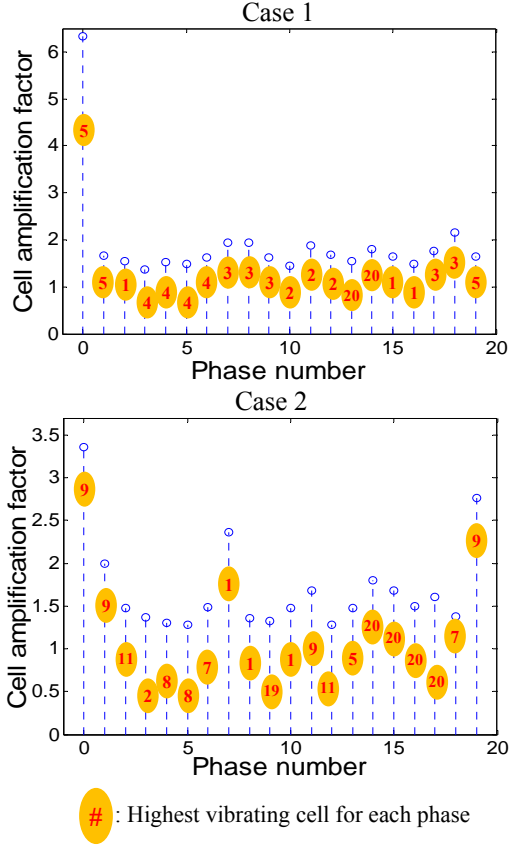


Figure 10: Cell amplification factor for cases 1 and 2 of parameter variations

In a battery pack, the failure of a single cell leads to the failure of the entire pack. Thus, identifying the cell that is most likely to fail is a key issue. The very high computational speed of the PROM developed in this work allows a very rapid identification of the cells most influenced by parameter variations. For that, a (traveling wave) excitation is applied at the center of each cell. The forced response is collected for all cells. A cell amplification factor (CAF) is defined as

$$CAF_k^j = \max_{i=1, \dots, 20} \left(\frac{\max_{\omega} |A_i^{\Delta p}|}{\max_{\omega} |A_i^{nominal}|} \right), \quad (5)$$

where ω is the frequency of the excitation, $A_i^{\Delta p}$ is the amplitude of the response of cell i of a battery with

parameter variations, and $A_i^{nominal}$ is the amplitude of the response of cell i of the battery with nominal parameters. The j and k indicate the case of variation and phase number.

Herein, cell amplification factors were calculated for cases 1 and 2 of parameter variations for a variety of forcing patterns with different cell-to-cell forcing phase (ϕ) difference as shown in Figure 10. These results highlight that the 5th and 9th cells suffer the largest amplification in their vibratory response due to case 1 and case 2 variations. For example, the largest amplification factor was 6.3 on the 5th cell for case 1 parameter variations when $\phi = 0$. This means that at least one cell had a forced response that was 630% higher than would be predicted if all the cells were assumed to be identical. This level of forced response increase could potentially lead to a significant reduction in battery fatigue life. Therefore, the initial results suggest that it may be important to account for parameter variations in the cells when predicting the structural response and fatigue life of HEV batteries.

As shown in Figure 10, the worst cell is different for each case. That means that the worst cell should be detected by a statistical analysis. We applied 10,000 separate cases of random variations in the elastic modulus of all 20 cells. The average cell amplification factor for cell i and excitation phase k is defined as

$$CAF_k^j (AVG) = \max_{i=1, \dots, 20} \left(\frac{(1/n_v) \sum_{j=1}^{n_v} \max_{\omega} |A_i^{\Delta p}|}{\max_{\omega} |A_i^{nominal}|} \right), \quad (6)$$

where n_v is total number of samples (variations), $n_v = 10,000$. Figure 11 shows the probability that cell i is the worst of all cells when the excitation has phase k .

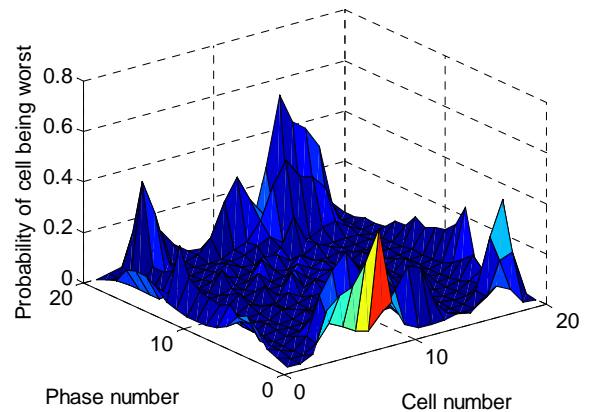


Figure 11: Probability of cell being worst based on 10,000 separate cases of random variations

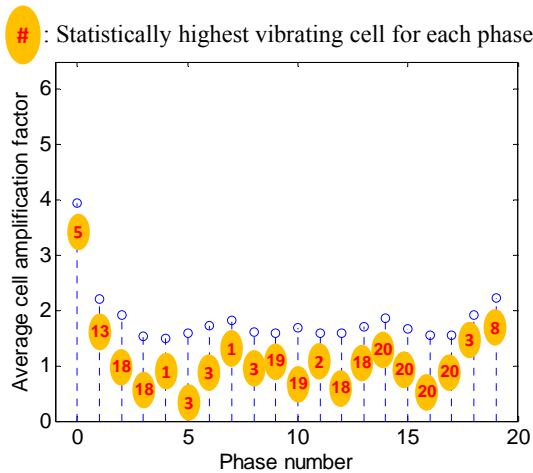


Figure 12: Average cell amplification factors for statistical analysis

Based on the statistical results, the average cell amplification factors for all cases applied are shown in Figure 12. The worst cell is the 5th cell. Its average cell amplification factor is 3.9.

CONCLUSIONS

The dynamic characteristics of HEV battery packs can be sensitive to small structural variations among battery cells because the system features high modal density. Thus, to predict the fatigue life, statistical calculations should be performed. However, a structural finite element model of a full HEV battery could easily have millions of DOF. The large model size makes it cumbersome or infeasible to run Monte-Carlo-type simulations.

In this paper, we developed new parametric reduced-order models (PROMs) to predict very quickly the structural dynamic response of HEV batteries. These PROMs are based on two key assumptions: (1) the mode shapes of the structure with variations can be represented as a linear combination of mode shapes of the structure with nominal parameters, and (2) the variability in parameters in the corresponding cell can be captured by mode shapes of the nominal cell with its boundary displaced the same amount as the frame.

As a numerical example, a PROM was generated for an academic model of a battery pack with 20 cells. The forced response results from the PROM were found to match very well with those from the full-order finite element model. The results also showed that small local variations in the structural parameters induce very large changes in the global response.

To help predict which cell is most likely to suffer fatigue failure, we defined an amplification factor that corresponds to the ratio of maximum forced response levels for the system with and without parameter variations. For the cases considered, the largest amplification factor was around 6.3. Thus, at least one cell had an increase in the forced response level of approximately 630% when small variations were included in the structural cell parameters, relative to the nominal system in which all cells are assumed to be identical. However, the worst cell may be different for each case of parameter variations. For that reason, 10,000 cases of random parameter variations were run and the average amplification factor was calculated for each cell. For the academic battery pack, the 5th cell was identified as the worst. The new computational methodology opens the door to the design of batteries with increased durability.

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